The Flow in Metals under Large Constant Stresses.

By E. N. da C. Andrade, B.Sc., Ph.D., late 1851 Exhibition Scholar of University College, London.

(Communicated by Prof. A. W. Porter, F.R.S. Received February 5,—Read May 7, 1914.)

§ 1. The object of this research was to examine the general laws of flow in metallic wires when extended in the region of large permanent set by stresses kept constant throughout the flow. Previously* I have investigated in detail the flow for one metal, lead, and put forward some empirical laws; it was desired to see if these laws could be extended to other metals, and especially to investigate the effect of temperature on the nature of the flow. For lead, rise of temperature causes a very rapid increase in the rate of the viscous† part of the flow (*loc. cit.*); hence it seemed likely that at very low temperatures the viscous part of the flow would cease altogether, although large permanent extensions might be obtainable, and thus lead might behave in this respect as iron behaves at atmospheric temperatures. Similarly iron at a high temperature might behave like lead at atmospheric temperatures. It was also desired to see if very pure metals behaved in the same way as commercial metals, for it has been supposed that the non-viscous character of the initial part of the extension-time curve is due to impurities. Further, the properties of the viscous flow itself were to be investigated in greater detail.

Investigations on these points are described in this paper; a summary of the results will be found in § 10. Incidentally, in the case of alloys, a type of flow not hitherto observed has been found.

§ 2. To test the flow under constant stress the device of the hyperbolic weight was again employed. By the use of such a weight, sinking as the wire stretches into a liquid of suitable density, the effective load is diminished in such a way that the stress over the cross-section remains constant as the wire thins (see *loc. cit.*). For experiments at lower temperature the wires were pulled upwards. A vertical open glass tube, the upper end of which is attached to a table, carries a bar at its lower end, to which the lower hook of the wire-holders is fastened; the upper end is pulled upwards by fine steel wires which pass over pulleys and carry the hyperbolic weight. Round this tube can be put any heating or cooling bath. For use with a sulphur bath the glass tube was replaced by a closed iron tube. The liquid employed for the hyperbolic weight, in the case of large loads, was mercury. For the higher temperatures an electric furnace was used, and the wire arranged horizontally. It was enclosed in a tube of combustion glass which projected some distance from the furnace at each end; the joints, where the wire-holders passed out of the tube, were made close with thin rubber, so that in cases where the specimen was to be kept clean for microscopic observation a stream of hydrogen could be maintained through the tube during the experiment. The temperature was measured with a thermocouple.

---

* E. N. da C. Andrade, 'Roy. Soc. Proc.,' A, vol. 84, p. 1 (1910).

† By viscous flow I refer throughout to that part of the flow for which the rate of extension, per unit length flowing, remains constant, or $\frac{1}{l}\frac{dl}{dt} = \text{constant}$, as would be the case for a very viscous liquid.

§ 3. The metals investigated were :—Lead, as being a metal which shows well at room temperature all the features of the flow in metals, viz., the immediate stretch, the β-flow, and the viscous flow; tin, as the second component in the alloy investigated; iron and copper, as showing very little viscous flow at room temperature; mercury, because it can be easily obtained very pure; and a lead-tin alloy and a brass to represent alloys, which were found to behave anomalously; a few experiments were also done on German silver, as a three-metal alloy.

All the metals, except mercury, were bought in the form of wires. For mercury two hollow glass hooks were connected to a thin glass tube with rubber, and the whole poured full of mercury and then immersed in a bath of solid carbon dioxide and alcohol. When the mercury was frozen solid the whole was removed from the bath, and the glass tube, previously scored with a diamond, quickly broken off; the mercury wire so obtained with a hook at each end was replaced in the bath, and subsequently stretched just like any other wire.

Series of observations were made at different temperatures with all the metals mentioned to obtain the law connecting extension with time, experiments at different stresses being performed at each given temperature. Constancy of stress in any one experiment was secured by means of a suitable hyperbolic weight.

The iron and copper wires were carefully annealed, and after this treatment gave consistent results. Lead anneals itself at room temperatures, that is, lead wires which have been left unstrained for a considerable time always give consistent results. Experiments on a piece of the lead wire used in 1909 gave results agreeing with those already published.

§ 4. It was found that without exception all the single metals—not alloys—gave at the various temperatures investigated curves of extension against time which could be closely represented by the formula

$$l = l_0(1+\beta t^{\frac{1}{3}})e^{kt},$$

given in the former paper, by assigning suitable values to the constants. Deviations of the calculated from the observed values in no case exceed, at any point of the range, 2 per cent. of the whole extension, and in most cases such deviations are less than 1 per cent. of the total extension, which, considering possible small variations from constancy of stress, is well within experimental error.* The final extensions obtained ranged up to about 20 per cent. of the original length for most metals and temperatures, but were occasionally larger, tin at room temperature being extended by 40 per cent. for instance.

The constants l_0, β, k are not purely artificial, but to each corresponds a physical process, and it would be impossible to get a curve to represent all the cases with less than three constants. l_0 represents the immediate length on loading,† β is the coefficient which gives a measure of that part of the flow whose rate decreases as the time increases, and k measures the final flow which proceeds viscously. The exponential form of the viscous term follows from the condition $\frac{1}{l}\frac{dl}{dt} =$ constant.

At low temperatures k is zero for all metals investigated even with the greatest stresses which could be applied without breaking the wire. β is very small at low temperatures for all stresses, probably zero if the temperature is taken low enough. l_0, however, can in general for larger stresses be obtained as great as at higher temperatures (without breaking the wire); thus lead wires loaded at −180° C. acquired considerable permanent set—up to 15 per cent. of the original length—and copper wires loaded at −78° C. were extended by similar amounts, but in such cases nearly the whole set takes place as soon as the load is applied. The very small subsequent flow does not become viscous with increasing time, but soon ceases to be perceptible. The form of curve for very low temperature is probably composed of a straight line of length l_0 along the axis of l, representing the immediate extension, and a straight line parallel to the time axis at a distance l_0. This is closely approached by lead at −180° C. and iron at 15° C. for all stresses.

As the temperature is taken higher the β-flow becomes more considerable, but there is still no viscous flow (*cf.* lead at −78° C., fig. 1). At a higher temperature we get the normal curve showing immediate extension, β-flow, and final flow of the viscous type. Considering the curves for different stresses at such a constant temperature, for increasing stress the constant β first increases slowly, then rapidly, and finally tends to a constant limiting value. k increases slowly at first, and then much more rapidly, as shown in the diagram representing the values of the constants for different stresses at constant temperature for lead (fig. 4). At a higher constant temperature the viscous flow becomes the preponderating factor; β again for increasing stress increases to a constant limit, which has the same value as for the lower temperature. k increases without limit. Thus with increasing temperature and stress β reaches a limit, while k increases finally very rapidly with both temperature and stress. The time-extension curves for lead, which are typical for all the single metals investigated, are given for the various temperatures in figs. 1, 2, and 3;* the variations of the constants of the formula with stress for the different temperatures are given in fig. 4. The constants for copper at 410° C. show the same features, β approaching a constant value, k increasing without limit.

---

* Deviations as large as 2 per cent. only occur in the earlier part of the curve, *i.e.* the first two minutes or so, where experimental errors are necessarily larger, owing to small unavoidable differences in the way in which the load is applied.

† Throughout the wires will be considered as of unit length before application of stress, thus $l_0 - 1$ represents the immediate extension.

The extension-time curves for mercury at $-78°$ C. are intermediate in type between those for lead at 160° C. and 17° C. The variation with stress could not be obtained, as, owing to the difficulty of annealing the mercury wire, we did not always get the same extensions on repeating with a fresh wire at the same stress; the type of curve, however, remained the same, and could always be represented by the formula. The mercury, being carefully distilled metal, is very pure, and these experiments show that the non-viscous form of the initial part of the curve is not in any way due to impurities in the metals.

The curves for iron at 444° C. resemble those for lead at 16° C.; there is no difference of type. Thus the behaviour of iron is not anomalous, as suggested by P. Phillips† from observations on small extensions at room temperature; it merely shows the low temperature type of flow, represented by lead at $-180°$ C., at ordinary temperature.

The main result of this part of the work is to show that typical metals of widely different nature obey the same general laws of flow, the apparent difference of type disappearing when a wide range of temperature is considered. The higher the temperature for a given metal the more the viscous type of flow predominates. Small impurities do not affect the general type of the extension curves; pure mercury at $-78°$ C. has much the same curves as soft commercial iron at 444° C.

---

* Only the initial portions of the curves are shown, the time of observation in many cases extended over hours.

† 'Phil. Mag.,' April, 1905, p. 513.

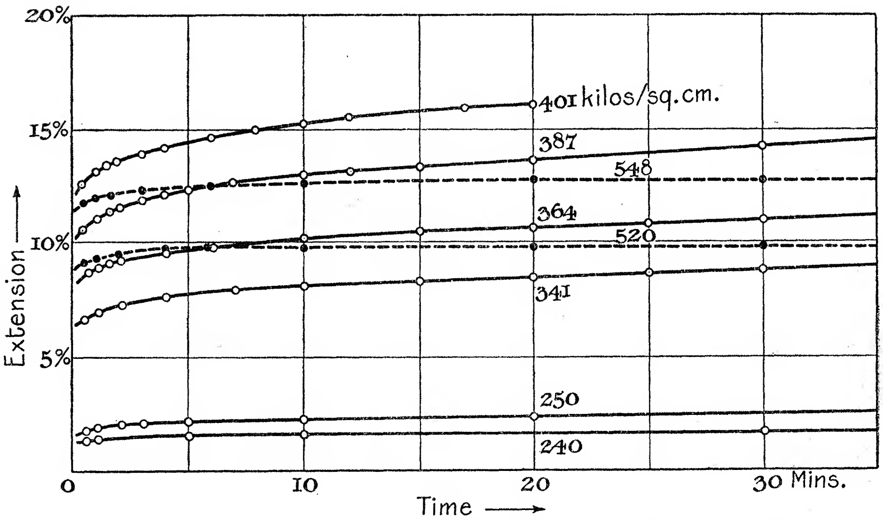

FIG. 1.—Lead at −180° and at −78° C. (The dotted curves refer to −180°, the continuous curves to −78° C.)

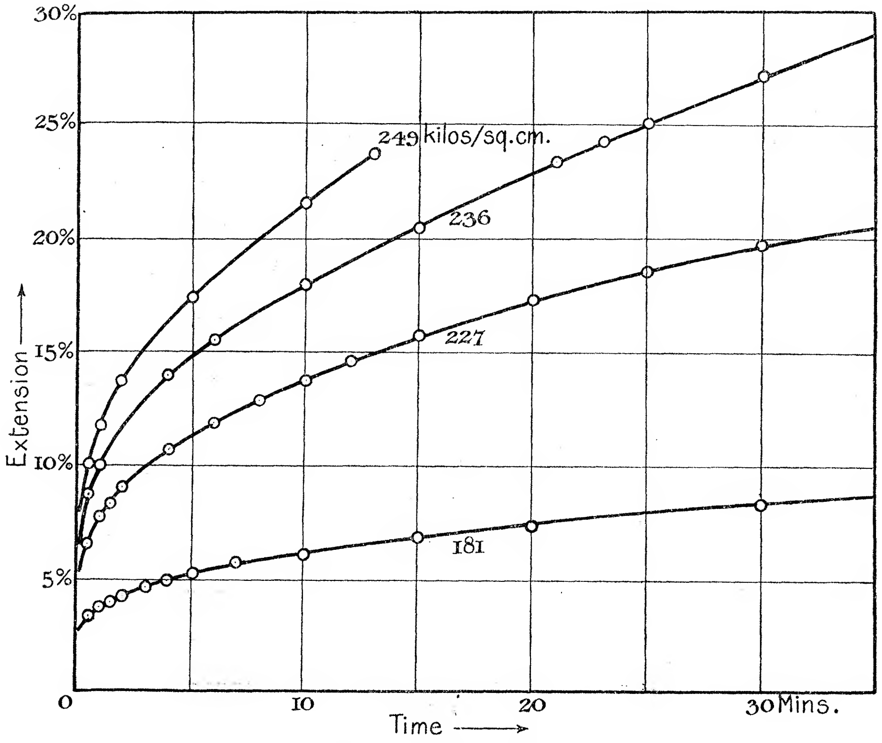

FIG. 2.—Lead at 17° C.

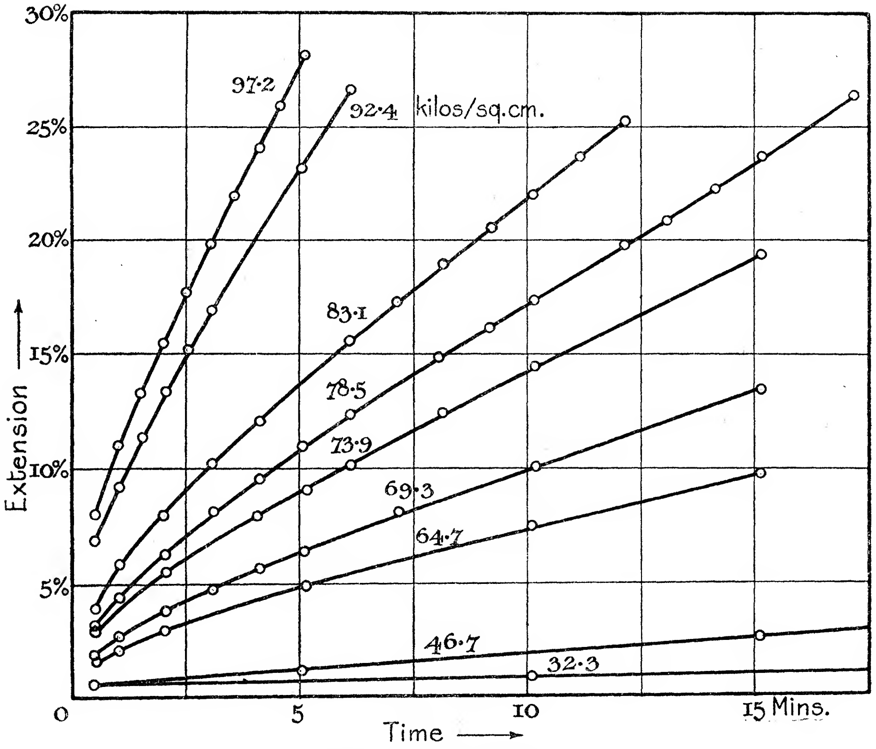

Fig. 3.—Lead at 160° C.

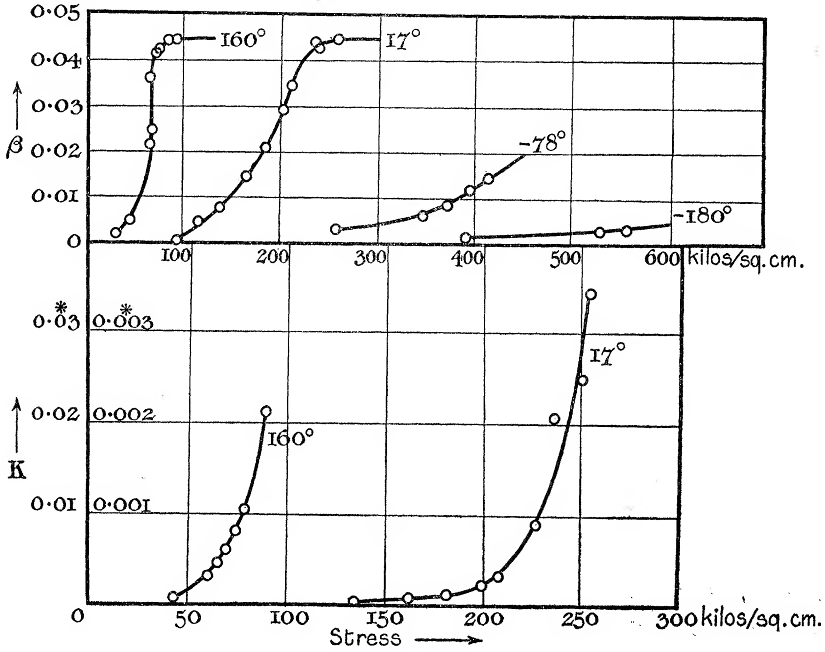

Fig. 4.—The Constants β and K for Lead at different Temperatures.

* *Note.*—The ordinates for the K curve at 160° are given on the left, at 17° on the right, of the axis of K.

The values of the constants for the different metals at the various temperatures and stresses are given in Table I on p. 336.

§ 5. When a lead-tin alloy (fuse wire, 21·5 per cent. lead, 78·5 per cent. tin) was investigated at room temperature it seemed to give curves which could be fitted by the law $l = l_0(1+\beta t^{\frac{1}{3}})e^{kt}$, the immediate extension l_0-1, and β being very small indeed, that is, the flow was of the viscous type almost from the start.* At −180° C. the alloy gave a curve resembling that for lead at this temperature, the extension nearly all taking place in the first moment; the extension could not, however, be obtained as great as for lead, the wire breaking first. But at an intermediate temperature, −78° C., the extension-time curves presented features not hitherto observed; there was very little immediate extension, the stretch started slowly, then became more rapid, and finally decreased again, the rate approaching a constant value, *i.e.*, the flow finally becoming viscous. The form of the curve is shown in the series of curves for fuse wire at different stresses at a temperature of −78° C. in fig. 5. The experiments on wires of pure tin at −78° C. showed that for

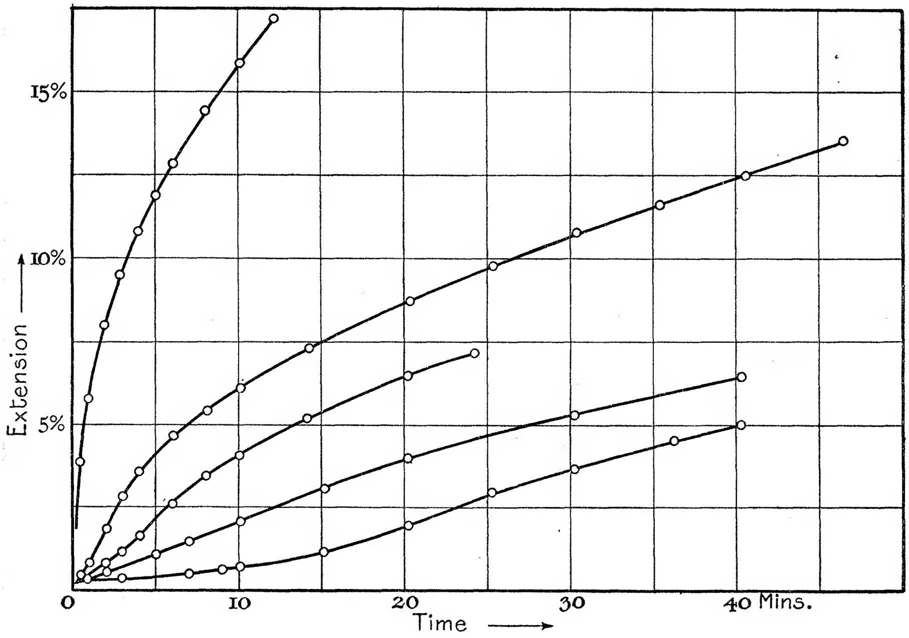

Fig. 5.—Lead-Tin Alloy at −78° C.

this metal the flow is of the type represented by the formula if $l_0 - 1$ and k are small; thus neither component of the alloy presents peculiarities at this temperature to account for the form of the curves. The alloy shows a

---

* See 'Roy. Soc. Proc.,' A, vol. 84, p. 5 (1910).

TABLE I.

Temperature.	Stress.	l_0.*	β.	k.
Lead. Area of cross-section (at 20° C.), 2·16 sq. mm.				
	kilos./sq. cm.			
−180° C.	388	1·050	0·0012	0
	520	1·091	0·0022	0
	548	1·118	0·0025	0
−78° C.	250	1·017	0·0026	0
	341	1·066	0·0060	0
	364	1·081	0·0085	0
	387	1·102	0·0115	0
	401	1·118	0·0142	0
17° C.	181	1·014	0·021	0·000115
	227	1·037	0·040	0·000890
	236	1·052	0·046	0·00190
	249	1·070	0·045	0·00350
160° C.	32·3		0·0020	0·00010
	46·7		0·0045	0·00095
	64·7		0·0210	0·0032
	69·3		0·024	0·0048
	73·9		0·041	0·0060
	78·5		0·041	0·0083
	83·2		0·043	0·0106
	92·4		0·043	0·0216
Copper. Area of cross-section (at 20° C.), 0·155 sq. mm.				
−78° C.	2548	1·143	0·002	0
15° C.	1696	1·037	0·0013	0
	1865	1·070	0·0014	0
	2287	1·1156	0·0021	0
410° C.	217		0	0
	522		0·0035	0·00018
	783		0·0080	0·00090
	957		0·0090	0·00240
	1044		0·0089	0·00460
	1131		0·0085	0·00680
Tin. Area of cross-section (at 20° C.), 0·915 sq. mm.				
20° C.	119	1·000	0·0066	0·0021
	191	1·000	0·0780	0·0042
−78° C.	321	1·000	0·0140	0
	386	1·0027	0·0225	0·00032
Iron. Area of cross-section (at 20° C.), 0·204 sq. mm.				
444° C.	956	1·0015	0·0005	0·00000
	1195	1·0030	0·0012	
	1435	1·0030	0·0037	
	1650	1·0030	0·0070	
	1920	1·0115	0·0123	0·00003
	2160	1·0170	0·0170	0·00008
	2400	1·0180	0·0210	0·00033

---

* The values of l_0 have not been determined accurately, as this would need special precautions. They are given, however, as some indication of the amount of immediate extension.

softening soon after application of stress, instead of the usual hardening; such a softening of a metal has not, as far as I am aware, been previously observed.

Curves of this form were sought with brass, as another two-metal alloy, and were obtained at temperatures of 350° C. and 400° C. At higher temperatures the flow was of the viscous type, almost from the start obeying the law $l = l_0 e^{kt}$, as with fuse wire at room temperature. For both these alloys the curves at low temperature were of the type of lead at $-$ 180° C.; only at low temperature under large stresses is there marked immediate stretch.

A few experiments were carried out with German silver as a three-metal alloy. At 565° C. and 455° C. the curves were of the purely viscous type, showing neither immediate extension nor β-flow; at 355° C. they were of the type of lead at $-78°$ C. No new features were discovered.

§ 6. It has been shown that for metals the flow tends to become in every case finally viscous (if it does not become zero, which may be considered as a special case with a zero coefficient k). The question arises as to what change is produced in the metal which sets it in a condition to flow viscously, and how the change can be effected. Lead was used for a series of experiments. It was found that by extending the wire rapidly under a large stress for a short time it can be put in a condition to flow viscously from the start when subject to a smaller stress. The more rapid the preliminary extension, assumed to be of the same amount in all cases, the smaller the rate of the viscous flow under the subsequent smaller stress; this rate, however, reaches a lower limit, after which increased rapidity of preliminary extension produces no further hardening, as tested by the smaller stress.

It might be supposed that the variation of k with stress at constant temperature does not express a physically simple relation because the wire, when the purely viscous flow sets in, is not in the same state for the smaller stresses as for the larger ones, it having been subjected to less preliminary strain in the former cases. Experiments were therefore carried out to ascertain the relation between the viscous rate and the stress when the wires were all subjected to the same treatment to bring them into a state such that, on being loaded, the flow was of the viscous type. A series of wires were all extended the same amount (about 18 per cent.) by the same stress; these wires were then subjected to various smaller stresses, under which they began to flow viscously almost from the start. The rate of viscous flow for the various stresses was plotted against the stress. The curve obtained was not very different from that of k against stress, in which case each stress is left to bring the wire to the viscous state of itself.

The curve is far from being linear. But it is not necessary to assume that the law of flow in the individual crystals of the metal is other than linear with the stress when the crystals have once been brought to flow viscously. An explanation of the form of the curve based on the assumption that the stress required to cause viscous flow of any single crystal depends upon its size will be given in § 9.

§ 7. It has also been found that severe preliminary twisting will put the wire in a state to give the flow viscous almost from the start. With increasing amount of preliminary twist the rate of flow first of all decreases, but reaches a minimum and then increases slowly, probably owing to tears in the metal. A twist of a given number of turns is more effective in hardening the metal if it is applied half in the one direction and then half in the other, than if it be applied all in the one direction. These experiments were carried out on lead, the twists causing permanent set. All this tends to show that any form of severe preliminary strain will condition the flow of the viscous type, and that with increasing severity of treatment the rate of this subsequent flow decreases to a constant final value.

§ 8. A series of microphotographs were taken of specimens of lead prepared by squeezing scraped lead wires between hot plate glass. The strips were extended by equal amounts, about 12 per cent., some at − 180° C., some at 16° C., and some at 160° C., to see if difference in the structure could be discovered. Others were taken of strips of soft armature iron extended at 16° C. and at temperatures from 400° C. to 600° C.; these were polished before the extension, which took place in an atmosphere of hydrogen. The structure was in all cases sufficiently revealed by the straining without etching.

The main object was to see if slip bands were formed equally at the different temperatures, and, in general, if the different types of flow were attended by different types of subsequent structure. The photographs of iron showed slip bands formed with equal distinctness and frequency at both the higher and the lower temperatures. Although in the one case the extension, equal in all cases, occupied some minutes, and in the other was produced all at once, it was not possible to distinguish between the photographs. With lead the specimens extended at 160° C. showed a somewhat coarser crystalline structure than those for lower temperatures. In this case, as the flow was to last some minutes, only small stresses were applied; a finer structure would perhaps be produced if the largest stresses possible were used. It was observed that at room temperature more rapid extension seemed to give a finer structure than one of equal magnitude less rapidly effected.

Owing to differences between individual specimens, and parts of the same specimen, small differences of structure may pass undetected; there is, however, no considerable difference to be observed in the formation of slip bands at the different temperatures. This accords with the theoretical considerations of the next paragraph. The formation of slip bands does not necessarily condition a viscous flow; it is merely a necessary preliminary, which may or may not be followed by such a flow, according to the temperature.

§ 9. The work of Quincke,[*] Beilby,[†], Ewing and Rosenhain,[‡] and Faust and Tammann[§] has thrown much light on the structure and the process of plastic yield of metals, and their results are connected with many of the effects described in this paper. Even in the purest metals there are well-defined crystals separated from one another by thin walls (Quincke's "Schaumwände") of non-crystalline material. These walls consist, in the case of pure metals, of an amorphous allotropic modification of the pure substance; in other cases they contain in addition such small impurities as may be present. Beilby has shown that mechanical working of the surface or internal displacements, such as the relative slipping of the lamellæ of which a crystal is composed, produce fresh layers of the amorphous material, which is much harder than the crystalline form.

We can on this explain the form of the extension-time curve, and the physical meaning of the constants l_0, β, k.

When the wire is loaded there is an immediate stretching due to a large number of slips between the crystals and the lamellæ of crystals best disposed for slipping, with a consequent formation of layers of hard amorphous material. According to observations of Ewing and Rosenhain (*loc. cit.*) and Faust and Tammann (*loc. cit.*), which have been confirmed by the author for lead, the first slip bands appear normal to the direction of pull, and accordingly it appears that lamellæ which have this direction slip easiest. Owing to the formation of the hard layers between such lamellæ a redistribution of internal stress in the wire ensues, and other crystals not so favourably disposed for slip are now subject to forces large enough to cause slip and consequent hardening. Such a process continues as long as new crystals are being subjected to forces sufficient to cause the lamellar breakdown. This slipping, after the first immediate extension, constitutes the β-flow, and if the amorphous phase, when formed, can at the given temperature support the applied stress without yielding, with the slipping of the last lamellæ the flow comes to an end. The final number of layers formed increases with increase of applied external stress, as all the internal stresses are proportionally increased and consequently exceed the minimum value required to produce slip for crystals not included for smaller stresses. It appears from the experiments described that there is a limit, however, to the amount of breaking up into lamellæ which can take place in a crystal after the immediate extension has occurred; the number of elementary crystals which can be subsequently produced by stress does not increase indefinitely. The number produced by a given constant stress increases to a limit with the time, and with increasing stress these limits increase to another limit, corresponding to the formation of all possible elementary crystals.* This is represented by the limit to which β tends with increasing stress. If the amorphous phase, although harder than the original crystals, can yet flow under the applied stresses it proceeds to do so wherever it has been formed linearly with the time, and gives the viscous or k flow. At low temperatures the amorphous material is too hard to flow. The different forms of curves are caused by the amorphous phase becoming softer relatively to the crystalline as the temperature rises.† There is obviously no limit to the rate of viscous flow. The theory will also account for the results on the effect of preliminary twisting and stretching (see §§ 6 and 7) in bringing the metal into a state to flow viscously. For such preliminary working results in the formation of the amorphous phase due to the breaking up of the crystals, so that the β-flow process has been already carried out; with increased working we should expect to reach a limit of hardening, when all possible amorphous layers have been formed. This is what has been found.

---

[*] For a summary of Quincke's work see G. Quincke, "Die Schaumstruktur der Metalle," 'Internat. Zeitsch. f. Metallographie,' III, 1, p. 23 (1912).

[†] 'Roy. Soc. Proc.,' A, vol. 72, p. 218 and p. 226; vol. 74, p. 462; vol. 79, p. 463; vol. 82, p. 599.

[‡] 'Phil. Trans.,' A, vol. 193, p. 353.

[§] Zeitsch. f. Physik. Chemie,' vol. 75, p. 108 (1911).

The non-linear character of the curve connecting viscous rate and stress, when the wire has in each case been initially brought to the same state (§ 6), can be explained by the differences of size which exist among the crystals in one piece of metal. While the forces due to the external load acting on different crystals must be assumed to be proportional to their cross-sections, the forces of frictional resistance are independent of these cross-sections. If each crystal requires the same minimum force to act on it before it can begin to flow viscously, and the rate is then proportional to the excess force, then flow will take place among the larger crystals of the wire for smaller stresses (applied to the wire as a whole) than are necessary to cause flow in the smaller ones. Increasing the stress causes a proportionate increase in the rate of flow of crystals already flowing, and in addition sets up the flow in fresh crystals. When practically all the crystals are flowing we should expect a linear increase of rate of flow with stress, to which the experimental curve actually approaches.

---

* Crystals which cannot be further broken up by stress. If viscous flow cannot take place at the given temperature, further increase of strain leads to tears in the metal and subsequent rupture.

† The effect of increase of stress is similar to that of increase of temperature, in that it increases the importance of the viscous flow relative to the β-flow ; see curves.

In the case of alloys, to a difference in type of structure, as revealed by microphotographs, corresponds the difference in type of flow which has been demonstrated. Duplex alloys, such as the lead-tin alloy employed, have a ribbed structure, bands of light component (tin) and dark component (eutectic) running in and out of one another and interlocking.* The structure is thus somewhat analogous to a ferro-concrete block containing a large number of connected iron bars. On the application of large stresses the ribs of the harder component (eutectic), corresponding to the iron, must be broken before any flow can take place; this breaking is followed by a more rapid flow which corresponds to the softening of the metal which takes place in the initial stages of the flow at suitable temperatures (lead-tin, 78° C.; brass, 400° C.). There is no evidence to be seen in Ewing and Rosenhain's photographs for the formation of large slip bands such as those in the single metals. Hence at high temperatures, when both components are soft, the flow takes place viscously from the start, there being very slight, if any, formation of the coarse bands which give rise to immediate yield and β-flow for the single metals.

It is not claimed for the formula $l = l_0(1+\beta t^{\frac{1}{3}})e^{kt}$ that it is more than an empirical formula representing the flow in single metals very closely in all cases. $t^{\frac{1}{3}}$ should probably be replaced by a function of t which approximates closely to $t^{\frac{1}{3}}$ until t becomes very large, but approaches a constant limit as t approaches ∞. Until we know more concerning the mechanism of the slip we cannot profitably attempt to calculate a theoretical form for the curve; it may, however, be suggested that the method of approaching the problem will have to differ from those previously attempted in being a statistical one, taking account of the different sizes and dispositions of the crystals.

We have thus brought evidence to show that the laws of flow in single metals and alloys are intimately connected with their structure; in all cases there are two components of quite different physical properties to be considered, and their variation in relative quantity and relative softness with the temperature accounts for the various experimental results obtained. Metals are not homogeneous, but they are essentially distinguished in respect of their flow from homogeneous substances not by their ability or inability to flow, which is simply a question of temperature, but by the laws which govern this flow as regards both time and stress.

---

* *Cf.* Ewing and Rosenhain, *loc. cit.*, figs. 47 and 48.

§10. *Summary of Results.*

(1) If wires of single metals be extended by large stresses, precautions being taken to keep the stress constant as the wire thins, the length at any time t can be expressed by the empirical formula

$$l = l_0 (1+\beta t^{\frac{1}{3}})\, e^{kt}.$$

(2) Wires of pure distilled mercury in the solid state obey the same law, showing that this behaviour is not due to impurities in the metals.

(3) The various forms of extension-time curves which are observed for different metals at room temperature can all be obtained with one metal (lead) by suitably choosing the temperature, the constants of the formula varying with the temperature.

(4) At a given temperature with increasing stress the constant β tends to a constant value; the constant k increases at a rate which itself increases to a constant value.

(5) From experiments on lead it appears that the limit to which β tends with stress does not increase with the temperature. The limit cannot, however, be attained at low temperatures, owing to the wire breaking first.

(6) Large preliminary strain, whether of extension or rotation, puts the wire in a state to flow viscously.

(7) Duplex alloys, which possess a microstructure quite different from that of single metals, give a correspondingly different form of extension-time curve; the wire softens at a certain stage of the flow, instead of continually hardening up to a limit, as do the single metals.

(8) On the basis of the co-existence of two different phases, the crystalline and the amorphous, the results have received a qualitative explanation.

It gives me much pleasure to offer my thanks to Prof. A. W. Porter, F.R.S., in whose charge the University College laboratory was during the time that I was engaged in this work, for many kindnesses. I have also to thank Prof. Cormack for allowing me to use the microphotographic outfit in his department, and Mr. E. H. Bate for assistance in taking microphotographs.